Solutions 08

MATH 16B GSI:TAO SU TU 10/24/2017

1. (a) $y' = \cos(x) - y$
Sol: $y' + y = \cos(x) \Rightarrow (e^x y)' = e^x \cos(x)$, so $e^x y = \int e^x \cos(x) dx = \frac{1}{2}(\sin x + \cos x)e^x + c \Rightarrow y = \frac{1}{2}(\sin x + \cos x) + ce^{-x}$.
(b) $(x^2+1)\frac{dy}{dx} + 3x(y-1) = 0, y(0) = 2$
Sol: $y' + \frac{3x}{x^2+1} y = \frac{3x}{x^2+1}$, so $A(x) = \int \frac{3x}{x^2+1} dx = \int \frac{3}{2(x^2+1)} d(x^2+1) = \frac{3}{2}\ln(x^2+1) + const$. Take $A(x) = \frac{3}{2}\ln(x^2+1)$, and I.F.$= e^{A(x)} = (x^2+1)^{\frac{3}{2}}$. So $((x^2+1)^{\frac{3}{2}} y)' = (x^2+1)^{\frac{3}{2}} \frac{3x}{x^2+1} = 3x(x^2+1)^{\frac{1}{2}}$, then $(x^2+1)^{\frac{3}{2}} y = \int 3x(x^2+1)^{\frac{1}{2}} dx = \frac{3}{2}(x^2+1)^{\frac{1}{2}} d(x^2+1) = (x^2+1)^{\frac{3}{2}} + c \Rightarrow y(x) = 1 + c(x^2+1)^{-\frac{3}{2}}$. By the initial condition $y(0) = 2$, so $2 = 1 + c \Rightarrow c = 1$. Thus, $y(x) = 1 + (x^2+1)^{-\frac{3}{2}}$.

2. Skipped. (Idea: Draw the constant solutions first.)

3. $P' = 90t + 810 + 6\%P$, $P(0) = 500$.

4. (a) $f' = k(100 - f), f(0) = 1$ for some positive constant k.
(b) $f' = kf(100 - f), f(0) = 1$ for some positive constant k.

5. Draw the solution curves of the differential equation $yy' + x = 0((x, y) \neq (0, 0))$.
Sol: Separate the variables: $yy' = -x \Rightarrow ydy = -xdx$. So $\int ydy = \int -xdx \Rightarrow \frac{1}{2}y^2 = -\frac{1}{2}x^2 + c$. That is, $x^2 + y^2 = 2c$. Since $(x, y) \neq (0, 0)$, $c > 0$. Now, we see $x^2 + y^2 = 2c$ describe the collection of concentric circles centered at the origin.